Journal of Applied Mechanics and Technical Physics, Vol. 43, No. 4, pp. 570–575, 2002

RAREFIED GAS FLOW IN A CHANNEL WITH SPECULAR-DIFFUSE BOUNDARY CONDITIONS FOR ARBITRARY KNUDSEN NUMBERS. ONSAGER RELATIONS

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UDC 533.72

The problem of a rarefied gas flow in a channel for arbitrary Knudsen numbers has been solved analytically for the first time in the case where the scattering of gas molecules on the channel walls can be described by specular-diffuse boundary conditions. The mean free path of gas molecules is assumed to be constant, i.e., the collision frequency is proportional to molecular velocity. The gas moves under the action of a streamwise temperature gradient. Exact relations for heat and mass fluxes and for mean-mass velocity are obtained. It is shown that the Onsager relations are valid within the entire range of Knudsen numbers in the problem of heat and mass transfer in a channel. The dependence of heat and mass fluxes on the Knudsen number (channel thickness) is analyzed. A comparison with available results is performed.

Introduction. Gas flows with various Knudsen numbers have been studied in many papers (see, e.g., [1] and the papers cited there). Approximate methods have been developed for solving various problems, where the Knudsen number varies within wide limits. Some papers have appeared lately, where the gas flow for arbitrary Knudsen numbers is analyzed by numerical methods [2, 3]. It should be noted, however, that no analytical solution valid for arbitrary Knudsen numbers has been obtained for some nontrivial kinetic problem, whereas such solutions are known for low Knudsen numbers and are widely used, for example, for evaluating the accuracy of numerical methods. The reason is that it is rather difficult to describe the gas behavior with Knudsen numbers of the order of unity.

The problem of gas flow in a channel under the action of a streamwise temperature gradient for arbitrary Knudsen numbers is solved analytically in the present paper. We consider the case of specular-diffuse boundary conditions on the channel walls with arbitrary accommodation coefficients. Loyalka et al. [4] solved the problem in a similar formulation by numerical methods, but the study was confined to the case of equal accommodation coefficients on both walls of the channel. In addition, Loyalka et al. [4] used the Boltzmann–Crook–Welander equation with a constant collision frequency of molecules, whereas we use the same equation with the collision frequency proportional to molecular velocity. This corresponds to the assumption about a constant (independent of velocity) mean free path, which is valid for the model where molecules are hard spheres.

Available numerical solutions of the problem of gas behavior in a channel describe only the first moments of the distribution function (mass velocity of the gas and heat flux). In this sense, the solutions are not complete, since one has to know the distribution function to obtain a complete description of gas behavior. Such a problem in a closed volume can be solved only using analytical methods, which yield an exact solution.

An analytical solution of the problem for arbitrary Knudsen numbers allows one to consider, at a qualitatively new level, the validity of the known Onsager relations in thermodynamics of irreversible processes [5, 6], which attracts attention of many researchers [7, 8]. This problem for gas flows in a channel was previously analyzed only by approximate and numerical methods [9–11]. At the same time, an analytical (exact) proof of the validity of the Onsager relations in a wide range of Knudsen numbers is of principal importance. An analytical proof for the case of a wide channel and a low Knudsen number can be found in [12]. Despite the low Knudsen number, taking into account the fluxes in the Knudsen layer adjacent to the surface was very important. In the present paper, the

Moscow Pedagogical University, Moscow 107005. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 43, No. 4, pp. 98–105, July–August, 2002. Original article submitted January 30, 2002.

validity of the Onsager relations in the case of a gas flow in a channel for arbitrary Knudsen number is proved for the first time using analytical methods.

1. Formulation of the Problem and Governing Equations. We consider the classical problem of gas motion and heat transfer in a channel under the action of temperature and pressure gradients [1–5] whose relative values are assumed to be small. This condition allows us to linearize the problem. We assume that the Knudsen number (Kn) equal to the ratio of the mean free path to the channel width L can acquire an arbitrary value.

We introduce a Cartesian coordinate system with the origin at the center of the channel and the plane yz parallel to the upper and lower walls. We assume that the mass and heat fluxes are parallel to the z axis and the relative temperature and pressure differences satisfy the inequalities $|\Delta T|/T \ll 1$ and $|\Delta p|/p \ll 1$. Here $\Delta T = T_2 - T_1$ and $\Delta p = p_2 - p_1$ (T_1, p_1 and T_2, p_2 are the gas temperature and pressure at the beginning and end of the channel, respectively). The mass flux of the gas (J_m) and the heat flux (J_Q) along the channel are proportional to the pressure and temperature differences and can be presented as [5]

$$J_m = -2L_{11}(aL)v\Delta p/p - 2L_{12}(aL)\Delta T/T^2, \quad J_Q = -2L_{21}(aL)v\Delta p/p - 2L_{22}(aL)\Delta T/T^2,$$

where $v = 1/\rho = 1/(nm)$ is the specific volume, a is the channel thickness, and L is the channel width (along the y axis).

We describe the gas behavior by the Boltzmann–Crook–Welander kinetic equation

$$\mu\,\frac{\partial\psi}{\partial x}\,\psi + \psi(x,\mu,c) = \frac{1}{c}\,g_p + \Big(c - \frac{5}{2c}\Big)g_t = \frac{3}{4}\int\limits_{-1}^{1}(1-\mu'^2)\,d\mu'\int\limits_{0}^{\infty}\exp(-c'^2)c'^5\psi(x,\mu',c')\,dc' \tag{1.1}$$

with the collision frequency of molecules proportional to their velocity [1]: $\nu(v) = v\lambda_0^{-1}$. Here g_p and g_t are the dimensionless pressure and temperature gradients, respectively, and λ_0 is the order of the mean free path of the molecules. We write the linearized distribution function in the form $f = f_0(1 + c_y\psi(x^*,\mu,c))$, where f_0 is the absolute Maxwellian, $\boldsymbol{c} = \sqrt{\beta}\boldsymbol{v}$, $\mu = c_x/c$, and $x^* = x/\lambda_0$ are dimensionless quantities (in what follows, the asterisk is omitted); $\beta = m/(2kT)$, and k is the Boltzmann constant.

For the function ψ, the specular-diffuse boundary conditions written for the total distribution function f correspond to the following conditions:

$$\begin{aligned} &\psi(b,\mu,c) = (1-q_2)\psi(b,-\mu,c), \qquad -1<\mu<0,\\ &\psi(-b,\mu,c) = (1-q_1)\psi(-b,-\mu,c), \qquad 0<\mu<1. \end{aligned} \tag{1.2}$$

Here $0 \leqslant q_1 \leqslant 1$ and $0 \leqslant q_2 \leqslant 1$ are the accommodation coefficients of the tangential impulse of molecules on the upper and lower walls, respectively, and $b = a/\lambda_0 \sim 1/\mathrm{Kn}$.

We express the mass and heat fluxes through the function ψ:

$$\sqrt{\beta}J_m = nm\pi^{-3/2}\int dx\,dy\int\exp(-c^2)c_y^2\psi\,d^3c,$$

$$\sqrt{\beta}J_Q = nkT\pi^{-3/2}\int dx\,dy\int\exp(-c^2)c_y^2\Big(c^2 - \frac{5}{2}\Big)\psi\,d^3c,$$

$$\int dx\,dy = \int dy\int\limits_{-a}^{a}dx = \frac{L}{\nu\sqrt{\beta}}\int\limits_{-b}^{b}dx', \qquad b = a\sqrt{\beta}\nu \sim \frac{1}{\mathrm{Kn}}.$$

The dimensionless mass and heat fluxes are written as

$$J'_m = \nu J_m/(4aLnkT), \qquad J'_Q = m\nu J_Q/(4aLnk^2T^2)$$

or, in terms of L_{ij},

$$J'_m = -\frac{v\nu}{2}\,L_{11}\,\frac{\Delta p}{p} - \frac{\nu}{2Tp}\,L_{12}\,\frac{\Delta T}{T}, \qquad J'_Q = -\frac{\nu}{2pT}\,L_{21}\,\frac{\Delta p}{p} - \frac{p\nu}{2\rho^2T}\,\frac{\Delta T}{T}.$$

We rewrite these equalities using the dimensionless Onsager coefficients L'_{ij}:

$$J'_m = -L'_{11}\,\frac{\Delta p}{p} - L'_{12}\,\frac{\Delta T}{T}, \quad J'_Q = -L'_{21}\frac{\Delta p}{p} - L'_{22}\frac{\Delta T}{T}, \quad L'_{12} = \frac{\nu}{2Tp}\,L_{12}, \quad L'_{21} = \frac{\nu}{2Tp}\,L_{21};$$

also we rewrite these equalities in terms of the function ψ:

$$J'_m = \pi^{-3/2} \int_{-b}^{b} dx' \int \exp(-c^2) c_y^2 \psi \, d^3c, \quad J'_Q = \pi^{-3/2} \int_{-b}^{b} dx' \int \exp(-c^2) c_z \Big(c^2 - \frac{5}{2}\Big) \psi \, d^3c.$$

Then, we have

$$J'_m = -S_{11}k_p - S_{12}k_t, \quad J'_Q = -S_{21}k_p - S_{22}k_t,$$

$$k_p = \frac{\partial \ln p}{\partial z'}, \quad k_t = \frac{\partial \ln T}{\partial z'}, \quad \frac{\Delta p}{p} = z'_0 k_p, \quad \frac{\Delta T}{T} = z'_0 k_t, \quad z'_0 = \frac{z_0}{\lambda_0}, \quad S_{ij} = L'_{ij} z'_0.$$

Here z'_0 and z_0 are the dimensionless and dimensional lengths of the channel.

We decompose the function ψ into two terms proportional to k_p and k_t, respectively: $\psi = \psi_p k_p + \psi_t k_t$. Then, the mass and heat fluxes become

$$-S_{12} = \pi^{-3/2} \int_{-b}^{b} dx' \int \exp(-c^2) c_y^2 \psi_t \, d^3c; \tag{1.3}$$

$$-S_{21} = \pi^{-3/2} \int_{-b}^{b} dx' \int \exp(-c^2) c_y^2 \Big(c^2 - \frac{5}{2}\Big) \psi_p \, d^3c. \tag{1.4}$$

We have to prove that $S_{12} = S_{21}$.

2. Analytical Solution of Creep Problems for Arbitrary Knudsen Numbers. Owing to division of the distribution function into two terms, problem (1.1), (1.2) is also divided into two problems.

1. *Problem of the Mass Flux*:

$$\mu \frac{\partial \psi_t}{\partial x} + \psi_t(x, \mu, c) + c - \frac{5}{2c} = \frac{3}{4} \int_{-1}^{1} (1 - \mu'^2) \, d\mu' \int_{0}^{\infty} \exp(-c'^2) c'^5 \psi_t(x, \mu', c') \, dc',$$

$$\psi_t(b, \mu, c) = (1 - q_2) \psi_t(b, -\mu, c), \qquad -1 < \mu < 0,$$

$$\psi_t(-b, \mu, c) = (1 - q_1) \psi_t(-b, -\mu, c), \qquad 0 < \mu < 1,$$

We seek the solution of this problem in the form $\psi_t = (c - 5/(2c)) h(x, \mu)$, where $h = \xi^+(x, \mu)\theta_+(\mu) + \xi^-(x, \mu)\theta_-(\mu) - 1$, $\theta_+(\mu) = 1$ for $0 < \mu < 1$, $\theta_+(\mu) = 0$ for $-1 < \mu < 0$, $\theta_-(\mu) = 1$ for $-1 < \mu < 0$, and $\theta_-(\mu) = 0$ for $0 < \mu < 1$; $\xi^\pm(x, \mu)$ are unknown functions that satisfy the homogeneous equation

$$\mu \frac{\partial \xi^\pm}{\partial x} + \xi^\pm(x, \mu) = 0 \tag{2.1}$$

and the boundary conditions

$$\begin{aligned} \xi_-(b, \mu) &= q_2 + (1 - q_2) \xi_+(b, -\mu), \qquad -1 < \mu < 0, \\ \xi_+(-b, \mu) &= q_1 + (1 - q_1) \xi_-(-b, -\mu), \qquad 0 < \mu < 1. \end{aligned} \tag{2.2}$$

From Eq. (2.1), we obtain

$$\xi^+(x, \mu) = \xi^+(-b, \mu) \exp(-(x + b)/\mu), \qquad 0 < \mu < 1; \tag{2.3}$$

$$\xi^-(x, \mu) = \xi^-(b, \mu) \exp(-(x - b)/\mu), \qquad -1 < \mu < 0. \tag{2.4}$$

Here $\xi^+(-b, \mu)$ and $\xi^-(b, \mu)$ are unknown functions. First, we find $\xi^+(-b, \mu)$. Substituting (2.3) into the first condition of (2.2), we find the function $\xi^-(b, \mu)$. Substituting the latter into (2.4), we obtain

$$\xi^-(x, \mu) = [q_2 + (1 - q_2) \xi^+(-b, \mu) \exp(2b/\mu)] \exp(-(x - b)/\mu). \tag{2.5}$$

Substituting (2.3) and (2.5) into the left and right sides of the first equality in (2.2), respectively, we obtain

$$\xi^+(-b,\mu) = \frac{q_1 + q_2(1-q_1)\exp(-2b/\mu)}{1-(1-q_1)(1-q_2)\exp(-4b/\mu)}.$$

Hence, the function $\xi^+(x,\mu)$ is constructed:

$$\xi^+(x,\mu) = \frac{q_1 + q_2(1-q_1)\exp(-2b/\mu)}{1-(1-q_1)(1-q_2)\exp(-4b/\mu)} \exp\Big(-\frac{x+b}{\mu}\Big).$$

In a similar manner, we obtain the function $\xi^-(b,\mu)$

$$\xi^-(b,\mu) = \frac{q_2 + q_1(1-q_2)\exp(2b/\mu)}{1-(1-q_1)(1-q_2)\exp(4b/\mu)}$$

and the solution of (2.4)

$$\xi^-(x,\mu) = \frac{q_2 + q_1(1-q_2)\exp(2b/\mu)}{1-(1-q_1)(1-q_2)\exp(4b/\mu)} \exp\Big(-\frac{x-b}{\mu}\Big).$$

Thus, we have constructed the complete function $h(x,\mu)$ and, hence, solved the problem of the mass flux. According to (1.3), the mass flux is expressed through the function h:

$$-S_{12} = -\frac{1}{4\sqrt{\pi}} \int\limits_{-b}^{b} dx' \int\limits_{-1}^{1} (1-\mu^2) h(x',\mu)\, d\mu. \tag{2.6}$$

2. *Problem of the Heat Flux*:

$$\mu\,\frac{\partial \psi_p}{\partial x} + \psi_p(x,\mu,c) + \frac{1}{c} = \frac{3}{4}\int\limits_{-1}^{1} (1-\mu'^2)\, d\mu' \int\limits_{0}^{\infty} \exp(-c'^2) c'^5 \psi_p(x,\mu',c')\, dc',$$

$$\psi_p(b,\mu,c) = (1-q_2)\psi_p(b,-\mu,c), \qquad -1<\mu<0,$$

$$\psi_p(-b,\mu,c) = (1-q_1)\psi_p(-b,-\mu,c), \qquad 0<\mu<1.$$

We seek the function ψ_p in the form $\psi_p = f(x,\mu) + (1/c - 2\alpha)h(x,\mu)$, where $\alpha = (3/16)\sqrt{\pi}$; the function h has been introduced previously. As a result, we obtain two problems.

Problem 1:

$$\mu\,\frac{\partial f}{\partial x} + f(x,\mu) = \frac{3}{4}\int\limits_{-1}^{1} (1-\mu'^2) f(x,\mu')\, d\mu',$$

$$f(b,\mu) = (1-q_2)f(b,-\mu), \quad -1<\mu<0, \qquad f(-b,\mu) = (1-q_1)f(-b,-\mu), \quad 0<\mu<1.$$

Problem 2 is not given here, since it coincides with problem (2.1), (2.2) considered above. Note, we need not solve problem 1, since, when the function ψ_p is substituted into the expression for S_{21}, the function f vanishes after integration with respect to c. Finally, we obtain

$$-S_{21} = -\frac{1}{4\sqrt{\pi}} \int\limits_{-b}^{b} dx' \int\limits_{-1}^{1} (1-\mu^2) h(x,\mu)\, d\mu. \tag{2.7}$$

3. Mass and Heat Fluxes. Onsager Relations. The right sides of expressions (2.6) and (2.7) are identical. Hence, we have $S_{12} = S_{21}$, i.e., the Onsager relation is valid. The value of $S_{12} = S_{21}$ is denoted as $S = S(b,q_1,q_2)$. Substituting the function h into (2.6) [or (2.7)], we obtain

$$S(b,q_1,q_2) = -\frac{2b}{3\sqrt{\pi}} + \frac{1}{4\sqrt{\pi}}\int\limits_{0}^{1} \mu(1-\mu^2)\Big(1-\exp\Big(-\frac{2b}{\mu}\Big)\Big) \frac{q_1+q_2+(q_1+q_2-2q_1q_2)\exp(-2b/\mu)}{1-(1-q_1)(1-q_2)\exp(-4b/\mu)}\, d\mu. \tag{3.1}$$

We consider the particular cases of Eq. (3.1). For $q_1 = q_2 = q$, we have

$$S(b,q) = -\frac{2b}{3\sqrt{\pi}} + \frac{q}{2\sqrt{\pi}}\int\limits_{0}^{1} \frac{\mu(1-\mu^2)(1-\exp(-2b/\mu))}{1-(1-q)^2\exp(-2b/\mu)}\, d\mu. \tag{3.2}$$

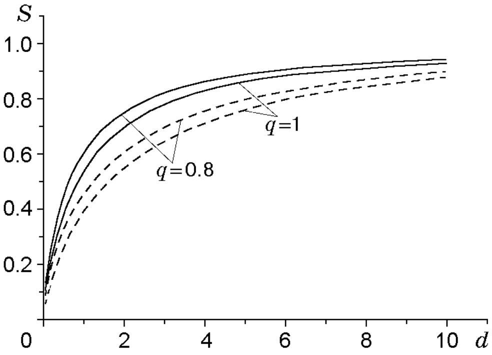

Fig. 1

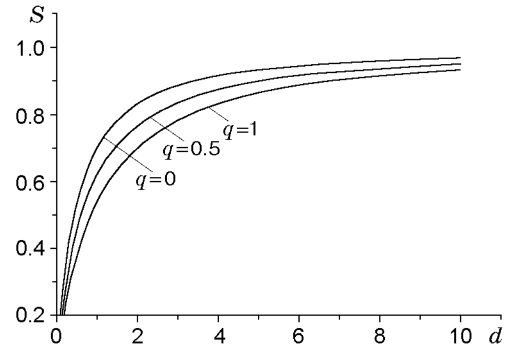

Fig. 2

It follows from Eq. (3.1) that the fluxes are equal to zero in the middle of the channel ($b = 0$). In the case of complete accommodation ($q_1 = q_2 = 1$), we have

$$S(b) = -\frac{2b}{3\sqrt{\pi}} + \frac{1}{2\sqrt{\pi}} \int_0^1 \mu(1-\mu^2)\Big(1 - \exp\Big(-\frac{2b}{\mu}\Big)\Big)\, d\mu.$$

It also follows from Eq. (3.1) that we have the same expression for the fluxes in the case of complete accommodation at one of the channel boundaries ($q_2 = 1$ and $q_1 = q$ or $q_1 = 1$ and $q_2 = q$):

$$S(b,q) = -\frac{2b}{3\sqrt{\pi}} + \frac{1}{4\sqrt{\pi}} \int_0^1 \mu(1-\mu^2)\Big(1 - \exp\Big(-\frac{2b}{\mu}\Big)\Big)\Big[1 + q + (1-q)\exp\Big(-\frac{2b}{\mu}\Big)\Big]\, d\mu. \qquad (3.3)$$

4. Profile of the Mean-Mass Velocity in the Channel. The profile of the mean-mass velocity in the channel can be constructed using the formula

$$U(x) = \pi^{-3/2} \int \exp(-c^2) c_y^2 \psi_t(x,\mu,c)\, d^3c.$$

Performing two internal integrations, we obtain

$$U(x) = -\frac{1}{4\sqrt{\pi}} \int_{-1}^1 (1-\mu^2)[-1 + \xi^+(x,\mu)\theta_+(\mu) + \xi^-(x,\mu)\theta_-(\mu)]\, d\mu.$$

Substituting the functions $\xi^{\pm}(x,\mu)$ found above into the latter expression, we obtain

$$U(x) = \frac{1}{3\sqrt{\pi}} - \frac{1}{4\sqrt{\pi}} \int_0^1 (1-\mu^2)[\xi^+(x,\mu) + \xi^-(x,-\mu)]\, d\mu$$

or, in an explicit form,

$$U(x) = \frac{1}{3\sqrt{\pi}} - \frac{1}{4\sqrt{\pi}} \int_0^1 \frac{(1-\mu^2)Q(x,\mu)}{1-(1-q_1)(1-q_2)\exp(-4b/\mu)} d\mu, \qquad (4.1)$$

where $Q(x,\mu) = q_1 \exp(-(b+x)/\mu) + q_2 \exp(-(b-x)/\mu) + q_2(1-q_1)\exp(-(3b+x)/\mu) + q_1(1-q_2)\exp(-(3b-x)/\mu)$. It should be noted that the first term in (4.1) is the coefficient of thermal creep (see [13, 14]) of the gas along a flat surface.

We determine the mean free path in accordance with [1]: $l = (\mu/p)\sqrt{\pi kT/(2m)} = 8\lambda_0/15$ (μ is the dynamic viscosity of the gas).

Figure 1 shows the mass flux as a function of the dimensionless thickness of the channel $d = 2a/l$; the solid and dashed curves are plotted using Eq. (3.2) and the data of [4], respectively. Figure 2 shows the mass flux versus the channel thickness d, which was constructed using Eq. (3.3). Figure 3 shows the profile of the mean-mass velocity in the channel, which was constructed using Eq. (4.1) for $q_1 = q_2 = q$ (solid curves) and the data of [4] (dashed curves). The mean-mass velocity profile constructed using Eq. (4.1) for $q_1 = 1$ is plotted in Fig. 4. In Figs. 3 and 4, we have $\xi = x/b$.

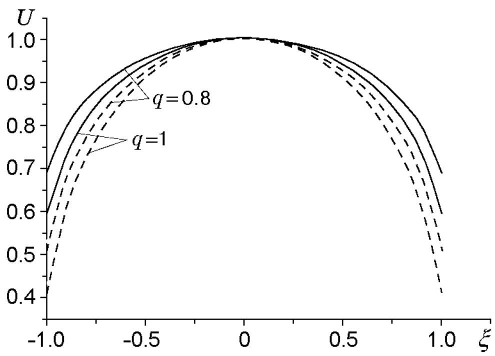

Fig. 3

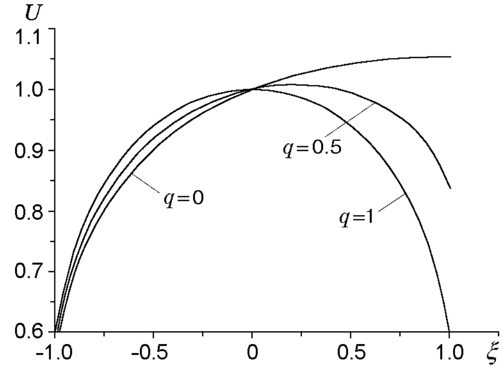

Fig. 4

It follows from Figs. 1–4 that the mass fluxes reach the asymptotic value ($d \to \infty$) more rapidly in the case of using the kinetic equation with the collision frequency proportional to molecular velocity than in the case of using the kinetic equation with a constant collision frequency. If the accommodation coefficients are different, the velocity profiles become asymmetric about the channel centerline.

5. Conclusions. The problem of heat and mass transfer in a channel filled by a rarefied gas with the collision frequency proportional to molecular velocity is solved exactly for the first time for arbitrary accommodation coefficients on the channel walls. The analytical results obtained are compared with numerical calculations [4].

It is shown that the Onsager relations are rigorously fulfilled for the problem considered within the entire range of Knudsen numbers.

This work was partly supported by the Russian Foundation for Fundamental Research (Grant No. 99-01-00336).

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